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13. ABSTRACT (Maximum 200 words)

This grant supported Anna Patterson from June 1994 to October 1996, Russ Allbery from October 1995 to March 1997, and Larry Yogman from January 1997 to June 1997. Patterson wrote (i) "A new semantics for constructible falsity", presented at ASL Summer Meeting '96, appearing in the Bulletin of Symbolic Logic and submitted to JSL; (ii) "Bisimulation and Propositional Intuitionistic Logic", presented at ESSLLI'96 and in more detail at Concur 1997 in Warsaw; and (iii) (with T. Costello) "Guilt-Free Exponentials", to be submitted. In addition she wrote the early stages of her thesis.

Allbery implemented a much faster version of Vineet Gupta's Chu space calculator. The one difficult operation is tensor product, which is known to be NP-complete and therefore requires good heuristics in order to be useful in practice. The bulk of the effort went into tuning for this operation.

Yogman reimplemented the Gupta-Allbery calculator as a Java applet. This allows the user of a Java-enabled web browser, from anywhere in the world, to download and start up the calculator with one click. This version is not as fast as Allbery's and therefore not as useful for testing hypotheses about large Chu spaces. However, as a pedagogical tool for studying Chu spaces it is excellent.

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Professor Vaughan R. Pratt

August 26, 1997

Technical Part

Students supported:

- Anna Patterson (June 1994 to October 1996)

- Russ Allbery (October 1995 to March 1997)

- Larry Yogman (January 1997 to June 1997)

1 Anna Patterson (June 1994 to October 1996)

Anna Patterson has carried out the following projects under this grant.
1. A new semantics for constructible falsity
2. Bisimulation and Propositional Intuitionistic Logic
3. Guilt-Free Exponentials
4. Thesis

1.1 A new semantics for constructible falsity

Anna Patterson (Stanford University)
March 1996

Constructive mathematics preserves the symmetry of Boolean logic's two truth values when it allows a proposition to be undecided: "could be either true or false". Intuitionistic logic destroys this symmetry by focusing on truth and offering only a nonconstructive "negation as failure", $\sim p = p \supset \bot$. Nelson's system N[1] of constructible falsity restores symmetry to intuitionistic logic with an involutory *strong* negation $\neg p$ asserting the actual falsity of p. (cf. Zaslavskiĭ[3] for another symmetrization.) Thomason[2] proposes

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a Kripke structure interpretation with three truth values for which he shows that N is sound and complete.

We give a Kripke model having only two truth values for which we show that N is sound and complete. Truth is defined on intervals (w_t, w_f) in the Kripke structure such that w_f is accessible from w_t. Truth is judged from one end w_t, falsity from the other w_f. A formula φ holds at an interval if and only if it holds at all non-trivial subintervals. We interpret strong negation as dualizing the Kripke structure with respect to both truth and accessibility.

Presented ASL Summer Meeting '96 to appear in the Bullitin of Symbolic Logic and Submitted to the JSL.

1.2   Bisimulation and Propositional Intuitionistic Logic

Anna Patterson (Stanford University)
June 1996

Brouwer suggests that $p \supset q$ can be interpreted as a computation that given a proof of p constructs a proof of q. Dually we can see this as saying that every counter model of q contains a counter model of p. However, what does it mean to contain another Kripke structure? We read this as: every counter model of q contains a substructure that is bisimilar to a counter model of p.

If q and p are interderivable, then the counter models contain each other. Thus we study *bisimulation* as a way of studying interderivability. Using bisimulation, we are able to characterize validity in a Kripke structure.

Theorem 1: Let K be a finite Kripke structure for propositional intuitionistic logic, then two worlds in K are bisimilar if and only if they satisfy the same set of formulas.

This theorem lifts to structures in the following manner.

Theorem 2: Two finite Kripke structures K and K' are bisimilar if and only if they have the same set of valid formulas.

This correspondence exists also for finite principal filter structures and saturated structures.

Observational and Logical Equivalence at ESSLLI '96. Fuller version appeared at Concur 1997, June, Warsaw .

1.3   Guilt-Free Exponentials

Anna Patterson and Tom Costello (Stanford University)

January 1997 (Submitted August 1996)

Girard introduces an operator ! called an exponential, to add structural rules to linear logic A formula ! A can be replicated or discarded. Thus the exponential removes the "linear" property of linear logic.

The exponential is also used to embed intuitionistic logic in linear logic. Girard suggests that intuitionistic implication $A \supset B$ can be represented as ! A –o B.

We suggest that these two uses can be separated. We look at how the second transformation can be carried out in $N-$ a logic which already contains all the structural rules of intuitionistic logic.

We show how exponentials arise from adding constants to the logic of constructible falsity $N-$. These two new units complete $N-$, in the sense that there is a representation theorem for $N-$ with these additional units onto $H \times H^{op}$ where H is a Heyting algebra.

ASL Winter Meeting

Full Version to appear at the World Congress on Paraconsistency and Minimal Logics August 1997 and Special Issue of Acta Informatica 1998.

1.4 THESIS

The Logic of Constructive Duality

Abstract

We present an investigation of duality in the traditional logical manner. We extend Nelson's symmetrization of intuitionistic logic, constructible falsity, to a self-dual logic — constructible duality. We develop a self-dual model by considering an interval of worlds in an intuitionistic Kripke model. The duality arises through how we judge truth and falsity. Truth is judged forward in the Kripke model, as in intuitionistic logic, while falsity is judged backwards.

We develop a self-dual algebra such that every point in the algebra is representable by some formula in the logic. This algebra arises as an instantiation of a Heyting algebra into several categorical constructions. In particular, we show that this algebra is an instantiation of the **Chu** construction applied to a Heyting algebra, the second Dialectica construction applied to a Heyting algebra, and as a posetal case of a suggestion by Pitts for the study of recursion and corecursion. Thus the algebra provides a common base for these constructions, and suggests itself as a necessary part of any logical treatment of constructive duality.

Implicit programming is suggested as a new paradigm for computing with constructible duality as its formal system. We show that all the operators that have computable least fixed points are definable explicitly and all operators with computable greatest fixed points are definable implicitly within constructible duality.

2 Russ Allbery (October 1995 to March 1997)

Russ rewrote Vineet Gupta's Chu space calculator to incorporate faster algorithms, particularly for tensor product. This version has proved extremely useful in testing many hypotheses about the behavior of Chu spaces in connection with our work of the past year on full completeness of multiplicative linear logic under the Chu space interpretation.

It is available to interested parties as a directory of C++ programs suitable for compilation under most Unix-like environments. It has compiled successfully under SunOS, Solaris SPARC, Solaris x86, and Linux.

3 Larry Yogman (January 1997 to June 1997)

Larry reimplemented the Gupta-Allbery calculator as a Java applet. This allows the user of a Java-enabled web browser, from anywhere in the world, to download and start up the calculator with one click. This version is not as fast as Allbery's and therefore not as useful for testing hypotheses about large Chu spaces. However as a pedagogical tool for studying Chu spaces it is excellent.

The online Chu space calculator can be accessed at
http://boole.stanford.edu/chucalc.html

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